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NUMERICAL INVESTIGATION OF EIGENOSCILLATIONS NEAR HONEYCOMB IN THE CIRCULAR CHANNEL

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ABSTRACT

Eigenvalue problem of $(-\Delta)$ operator with the finite-energy and Neumann conditions are investigated . The classification of possible eigenoscillations is carry out by the theory of group presentation. These modes of eigenoscillations are proved to exist and their quantity is found. Eigenvalues are studied numerically.

STATEMENT OF PROBLEM

A system of two equal strips forming a cross in the infinite channel of circular cross-section is considered. The line of strips' intersection divides it to half-and-half. All notations are shown in figure 1. Eigenoscillations with harmonic time dependence is assumed.

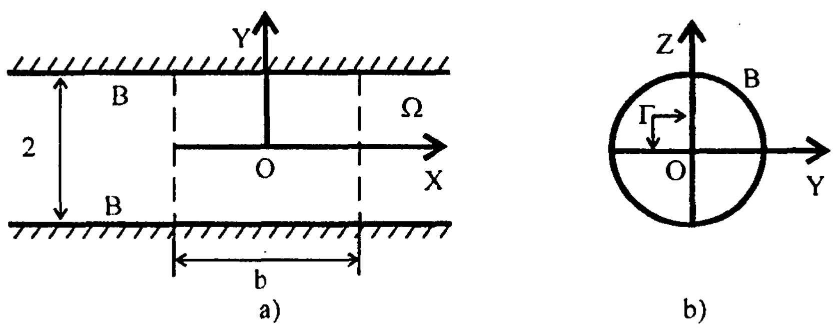

Fig. 1. Geometrical parameters of system of two strips forming a cross: a) top view; b) cross-section.

Mathematical statement of problem for potential function $u(x,y,z)$ is:

$$\begin{cases} \Delta u + \lambda^2 u = 0 & in\, \Omega / B / \Gamma \quad - \quad wave\ equation \\ \dfrac{\partial u}{\partial \vec{n}} = 0 & on\, B \cup \Gamma \quad - \quad Neumann\ condition \\ \int_{\Omega_0} \left(u^2 + (\nabla u)^2\right) d\Omega_0 < \infty & \Omega_0 \subset \Omega \quad - \quad finite-energy\ condition \end{cases} \qquad (1)$$

CLASSIFICATION AND EXISTENCE OF EIGENOSCILLATIONS

The self-adjoint extension of $(-\Delta)$ has positive continuous spectrum. The main difficulties is that discrete spectrum of problem (1) is imbedded in continuous spectrum of operator $(-\Delta)$.

Symmetry group of the cross in the channel contains subgroups D_4, C_4 (rotation on $\pi/2$ in the plane parallel to OYZ) and D_1^{OYZ} (mirror symmetry with respect to plane OYZ) [1]. Subgroup D_1^{OYZ} means that the admissible solution space can be decomposed into solutions, which are odd and even at variable x.

Generating elements of subgroup D_4 are r and s, where r is rotation at $\pi/4$ with respect to axis OX and s is mirror symmetry with respect to plane OXZ. Irreducible representations of subgroup D_4 are shown in the table 1.

	ψ_1	ψ_2	ψ_3	ψ_4
r^k	1	1	$(-1)^k$	$(-1)^k$
sr^k	1	−1	$(-1)^k$	$(-1)^{k+1}$

Table 1. Irreducible representation of D_4, where ψ_i, $i = 1,..,4$ are their characters.

Irreducible representations τ_k of group C_4 are $\tau_k\left(C_4^m\right)=\exp(i\pi km/2)$, where $k,m=0,1,2,3$, so the admissible solution space can be decomposed into solutions, which have next property: $C_4 < u(x,y,z) >= \exp(i\pi j/2)\cdot u(x,y,z)$, where $j = 0,..,3$. Oscillations with j=3 and j=1 are identical, they are equal to a wave moving clockwise and anticlockwise respectively.

Note that even eigenoscillations with respect to variables y, z can't exist [1]. So there are only 3 independent mode of oscillations (without consideration of evenness/oddness at x): 1) mode corresponds to ψ_2 (it will be called α-mode); 2) traveling wave at $j=1$ (β-mode); 3) mode corresponds to ψ_4 (γ-mode).

Let $\mu_{n,k}$ denote k-th root of equation $\left(J_n(x)\right)'=0$, $J_n(\mu_{n,k})\neq 0, k\in\mathbb{N}$. Let σ_0^2 be the point of the continuous spectrum of $(-\Delta)$. Value σ_0^2 for α, β, γ-modes is equal to $\mu_{4,1}^2, \mu_{1,1}^2, \mu_{2,1}^2$ respectively. Now it is possible to investigate discrete spectrum of problem (1) located below σ_0^2.

Theorem 1. α, β, γ-modes of the eigenoscillations always exist independent of geometrical parameters of the cross and the channel.

Lemma 1. The α-mode eigenfrequencies belong to interval $(\mu_{2,1}, \mu_{4,1})$.

Lemma 2. Quantity K of eigenvalues located below the cut-off of problem (1) satisfies to next inequalities <u>for α-mode:</u> $\max(1, b\sqrt{\mu_{4,1}^2-\mu_{2,1}^2}/\pi - 1)\leq K< b\sqrt{\mu_{4,1}^2-\mu_{2,1}^2}/\pi + 1$; <u>for β-mode:</u> $\max(1, b\,\mu_{1,1}/\pi - 1)\leq K< b\,\mu_{1,1}/\pi + 1$; <u>for γ-mode:</u> $\max(1, b\,\mu_{2,1}/\pi - 1)\leq K< b\,\mu_{2,1}/\pi + 1$.

Theorem 2. Finite-energy condition in any vicinity of plate edge is equivalent to next conditions (in cylindrical coordinate system chosen along a plate edge):

1) at the middle of the plate edge $u(b/2+x,\rho,\varphi) \simeq f(x) + \rho\cos(\varphi)\, g(x)$;
2) at other points of plate edge $u(b/2+x,\rho,\varphi) \simeq f(x) + \sqrt{\rho}\cos(\varphi/2)\, g(x)$,

at $\rho\to 0$, $f(x), g(x)\in W_2^1(\mathbb{R})$.

Lemma 3. α- and γ-modes have finite energy in any vicinity of plates edges. Finite-energy condition in any vicinity of plates edges for β-mode is equivalent to next conditions ($\forall m_1 \in \mathbb{N}$):

$$\sum_{n,m=1}^{+\infty} b_{n,m}(-1)^n e^{-b\gamma(2n-1,m)/2} \int_0^1 r J_{2n-1,m}(\mu_{2n-1,m} r) J_{1,m_1}(\mu_{1,m_1} r)\,dr = 0.$$

NUMERICAL INVESTIGATION

The linear infinite system (2) for coefficient of solution's expansion in domain $\Omega \cap \{x \geq 0,5b\}$ is obtained by sewing method

$$\sum_{n,m=1}^{+\infty} b_{n,m} \cdot \left((\gamma+\gamma_1)e^{b\gamma_1/2} + (-1)^l(\gamma-\gamma_1)e^{-b\gamma_1/2}\right)\frac{f(n)}{f^2(n)-g^2(n_1)} \times$$
$$\times \int_0^1 r J_{f(n)}\left(\mu_{f(n),m} r\right) J_{g(n_1)}\left(\mu_{g(n_1),m_1} r\right) dr = 0 \quad \forall m_1, n_1 \in \mathbb{N}. \tag{2}$$

where $\gamma(n,m) = \sqrt{\pi^2(n^2+m^2)-\lambda^2}, \gamma = \gamma\left(f(n),m\right), \gamma_1 = \gamma\left(g(n_1),m_1\right)$; for α-mode: $f(n)=4n, g(n_1)=4n_1-2$; for β-mode: $f(n)=2n-1, g(n_1)=2n_1-2$; for γ-mode: $f(n)=4n-2, g(n_1)=4n_1-4$; and l is even (odd) for even (odd) oscillations by x.

The system (2) was reduced to square and triangular partial sums, which were studied numerically. There are good coincidence between them. Figure 2 shows the variation of the eigenvalue λ with the cross-length b received by the first method.

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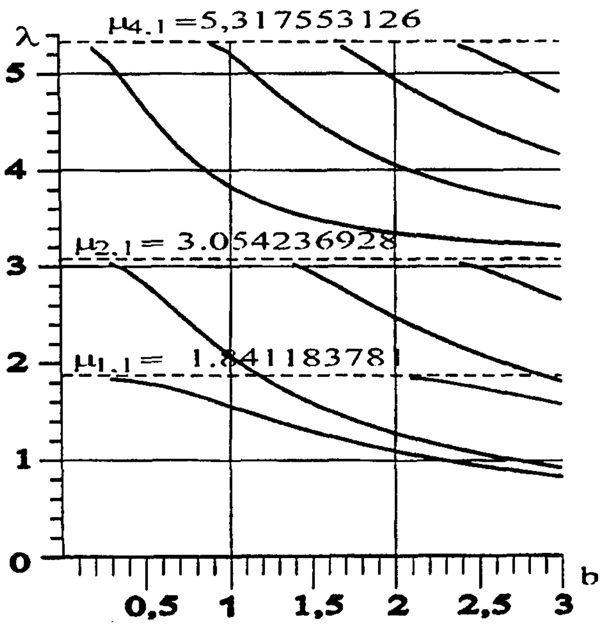

Fig. 2. Variation of the eigenvalue λ with the cross-length b.

RESULTS

1. It is proved that eigenoscillations always exist.
2. It is founded the quantity of mode of oscillations.
3. The diagrams eigenvalues against length of cross are obtained.

REFERENCES

[1] A.I. Makarov. Eolian tones of the unit part of the honeycomb. //Journal of Applied Mechanics and Technical Physics. Vol. 43, No. 5, 2002 (in Russian).